## Divide and Conquer example: Matrix Multiplication

The normal procedure to multiply two $n \times n$ matrices requires $n^{3}$ time. We could improve the required running time by the following Strassen's matrix multiplication algorithm.
Given two $n \times n$ matrices $A$ and $B$. Their product $C$ is also an $n \times n$ matrix. Assuming that $n$ is power of 2 . We can divide each of $A, B$, and $C$ into four $n / 2 \times n / 2$ matrices, rewriting the equation $A \times B=C$ as follows.

$$
\left[\begin{array}{ll}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{array}\right]\left[\begin{array}{ll}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{array}\right]=\left[\begin{array}{ll}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{array}\right]
$$

where let
$m_{1}=\left(A_{12}-A_{22}\right)\left(B_{21}+B_{22}\right)$
$m_{2}=\left(A_{11}+A_{22}\right)\left(B_{11}+B_{22}\right)$
$m_{3}=\left(A_{11}-A_{21}\right)\left(B_{11}+B_{12}\right)$
$m_{4}=\left(A_{11}+A_{12}\right) B_{22}$
$m_{5}=A_{11}\left(B_{12}-B_{22}\right)$
$m_{6}=A_{22}\left(B_{21}-B_{11}\right)$
$m_{7}=\left(A_{21}+A_{22}\right) B_{11}$
Then compute the $C_{i j}$ by the formulas
$C_{11}=m_{1}+m_{2}-m_{4}+m_{6}$
$C_{12}=m_{4}+m_{5}$
$C_{21}=m_{6}+m_{7}$
$C_{22}=m_{2}-m_{3}+m_{5}-m_{7}$
Assume that each scalar arithmetic operation takes constant time. Please write down the running time recurrence for the algorithm and derive its running time.

