## Divide and Conquer example: Matrix Multiplication

The normal procedure to multiply two  $n \times n$  matrices requires  $n^3$  time. We could improve the required running time by the following Strassen's matrix multiplication algorithm.

Given two  $n \times n$  matrices A and B. Their product C is also an  $n \times n$  matrix. Assuming that n is power of 2. We can divide each of A, B, and C into four  $n/2 \times n/2$  matrices, rewriting the equation  $A \times B = C$  as follows.

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

where let

 $m_{1} = (A_{12} - A_{22})(B_{21} + B_{22})$   $m_{2} = (A_{11} + A_{22})(B_{11} + B_{22})$   $m_{3} = (A_{11} - A_{21})(B_{11} + B_{12})$   $m_{4} = (A_{11} + A_{12})B_{22}$   $m_{5} = A_{11}(B_{12} - B_{22})$   $m_{6} = A_{22}(B_{21} - B_{11})$   $m_{7} = (A_{21} + A_{22})B_{11}$ 

Then compute the  $C_{ij}$  by the formulas

$$C_{11} = m_1 + m_2 - m_4 + m_6$$
  

$$C_{12} = m_4 + m_5$$
  

$$C_{21} = m_6 + m_7$$
  

$$C_{22} = m_2 - m_3 + m_5 - m_7$$

Assume that each scalar arithmetic operation takes constant time. Please write down the running time recurrence for the algorithm and derive its running time.