Q1(10 points): Asymptotic Notations

(a) (3 points) Which one of the following is first wrong statement?
1. $\Theta(n) + O(n) = \Theta(n)$
2. $\Theta(n) + O(n) = O(n)$
3. $\Theta(n) + \Omega(n) = \Theta(n)$
4. $f(n) = o(g(n))$ implies $g(n) = \Omega(f(n))$

(b) (7 points) Try to use the basic definition of $\Theta$-notation to show $n^2 - 10 \log_2 n = \Theta(n^2)$. 


• Q2 (12 points): Divide-and-Conquer
Suppose that a computer does not know how to apply dynamic programming techniques
to compute a function \( f(n) \), but it knows how to use the divide and Conquer approach
to compute \( f(n) \) as follows. The computer takes only constant time for scalar arithmetic
operations.

\[
f(n) = \begin{cases} 
0 & \text{if } n = 0 \\
1 & \text{if } n = 1 \\
f(n - 1) + f(n - 2) + n \log n & \text{if } n > 1 
\end{cases}
\]

(a) (8 points) Please write down the three steps of Divide, Conquer and Combine to describe
how the computer calculates \( f(n) \).

Divide: Do nothing.
Conquer:

Combine:

(b) (4 points) Please write down the running time recurrence if \( f(n) \) is computed using the
above approach.
Q3 (24 points): Recurrences

(a) (8 points) Given a recurrence $T(n) = 3T(n - 1) + 1$, please draw the recursion tree and derive a tight bound of $T(n)$. 
(b)(8 points) Given a recurrence $T(n) = 2T(n - 1) + n$, please use the substitution method to verify $T(n) = O(2^n)$.

Hint: use the hypothesis $T(n) \leq c(2^n - n)$ for some $c > 0$.

(c)(8 points) Please solve the recurrence $T(n) = 2T(n - 1) + n^2$ using the Master Method.

Hint: try to transfer the equation to another form and then solve it.
• Q4(24 points): Dynamic programming

(a)(9 points) For a Matrix-Chain problem with 4 matrices $A_1, A_2, A_3$ and $A_4$, please construct and draw the two tables as in the book if the dimension vector for these four matrices is $<3,1,5,4,2>$. 

(b)(3 points) Based on the tables in (a), what is the optimal parenthesization for the product $A_1A_2A_3A_4$?
(c)(9 points) For a LCS (longest common subsequence) problem with two input sequences $X = < C, A, A, B, B, D, C >$ and $Y = < C, B, A, D, B, B, C >$, please draw the table(s) as in the book.

(d)(3 points) Based on the table(s) in (c), what is the longest common subsequence for $X$ and $Y$?