CS 321-001 Data Structures (Spring 2019)

Homework #1 (80 points), Due Date: 2/19/2019 (Tuesday)

- **Q1(12 points): Asymptotic Notations**

  (a)(4 points) Which one of the following is a wrong statement?

  1. $\Theta(n) + 5\log n^2 = \Omega(n)$
  2. $5n + \Omega(n) = \Theta(n)$
  3. $\Theta(n) + \Omega(n) = \Omega(n)$
  4. $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$

  (b)(4 points) Which one of the following sorting algorithms will have the worst best-case running time?

  1. Selection sort
  2. Insertion sort
  3. Heap sort
  4. Quick sort

  (c)(4 points) Explain why the statement, "The running time of an algorithm is $\Omega(1)$," is meaningless.

- **Q2(18 points): Running Time and Growth of Functions**

(a)(10 points) Assume evaluating a function $f(n)$ in the pseudocode below takes $\Theta(\log n)$ time.

```
i = 1;
sum = 0;
while (i <= n)
      do if (f(i) > k)
            then sum += f(i);
         i = i+1;
```

What is the running time (use an asymptotic notation) of the above code? Justify your answer.

(b)(8 points) For the following functions, please list them again but in the order of their asymptotic growth rates, from the least to the greatest. For those functions with the same asymptotic growth rate, please underline them together to indicate that.

$$2^n,\ (\log_2 n)^n,\ n!,\ (\log_2 n^n),\ n^2,\ (\log_2 n^{10}),\ n \log_e n,\ 3^n, \log_{10} n$$

- **Q3(26 points): Sorting**

(a)(7 points) For a given input array $A :< 1, 5, 2, 4_a, 6, 4_b, 9, 11, 10 >$, what is the sequence of numbers in A after calling Build-Max-Heap(A) ? (please show the intermediate trees).

(b)(7 points) For a given input array $A :< 7_a, 6, 7_b, 1, 3, 10, 8, 4, 5 >$, what is the sequence of numbers in A after the first partition (by calling `Partition(A, 1, 9)`)? Note that 1 and 9 in `Partition(A, 1, 9)` function call are array indexes.

(c)(7 points) By using the MaxHeap data structure to implement a priority queue, some applications may need to change the data (priority) of a specific node i. That is, given an index i, change the priority of node i to a new priority t. Please write a pseudocode for this procedure. You can implement the procedure by calling the `MaxHeapifyDown(A, i)` and/or `MaxHeapifyUp(A, i)` methods.

```
Max-Heap-update(A, i, t)
{

}
```

(d)(5 points) Please describe how to use a priority queue to implement a queue abstract.

- **Q4(14 points): Linear Time Sorting**

  (a)(6 points) Please describe the reason(s) why we choose the `counting sort` algorithm to sort each digit in the `Radix Sort`?

  (b)(8 points) What is the best running time to sort n integers in the range $[0, n^3]$, and How?

- **Q5(10 points): Basic Data Structures - Stacks and Queues**

  (a)(10 points) How to use two queues to implement a stack so that `push` runs in $O(n)$ and `pop` runs in $O(1)$? Suppose the queues have no size limit. Please describe your algorithm without pseudocode.