Static Allocation, Demand Paging

- Number of page frames is static over the life of the process
- Fetch policy is demand
- Since $S_t(m) = S_{t-1}(m) \cup \{r_t\} - \{y\}$, the replacement policy <u>must choose y</u> -- which uniquely identifies the paging policy

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0																
1																
2																

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	$\underline{2}$	2	2													
1		$\underline{0}$	0													
2			$\underline{3}$													

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2												
1		0	0	1												
2			3	3												

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2										
1		0	0	1	1	1										
2			3	3	3	0										

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	3									
1		0	0	1	1	1	1									
2			3	3	3	0	0									

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	3	3	3							
1		0	0	1	1	1	1	1	1							
2			3	3	3	0	0	0	2							

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	3	3	3	0						
1		0	0	1	1	1	1	1	1	1						
2			3	3	3	0	0	0	2	1						

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2	2	2	2	<u>3</u>	3	3	<u>0</u>	0					
1		<u>0</u>	0	<u>1</u>	1	1	1	1	1	1	<u>3</u>					
2			<u>3</u>	3	3	<u>0</u>	0	0	<u>2</u>	1	1					

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	3	3	3	0	0	0	0			
1		0	0	1	1	1	1	1	1	1	3	3	6			
2			3	3	3	0	0	0	2	1	1	1	1			

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	3	3	3	0	0	0	0	4		
1		0	0	1	1	1	1	1	1	1	3	3	6	6		
2			3	3	3	0	0	0	2	1	1	1	1	1		

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	3	3	3	0	0	0	0	4	4	
1		0	0	1	1	1	1	1	1	1	3	3	6	6	5	
2			3	3	3	0	0	0	2	1	1	1	1	1	1	

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	3	3	3	0	0	0	0	4	4	7
1		0	0	1	1	1	1	1	1	1	3	3	6	6	5	5
2			3	3	3	0	0	0	2	1	1	1	1	1	1	1

Random Replacement

- Replaced page, y, is chosen from the m loaded page frames with probability 1/m

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	3	3	3	0	0	0	0	4	4	7
1		0	0	1	1	1	1	1	1	1	3	3	6	6	5	5
2			3	3	3	0	0	0	2	1	1	1	1	1	1	1

13 page faults

- No knowledge of ϖ ⇒ not perform well
- Easy to implement

Belady's Optimal Algorithm

- Replace page with maximal forward distance: $y_t = \max_{xeS\ t-1(m)} FWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2													
1		0	0													
2			3													

Belady's Optimal Algorithm

- Replace page with maximal forward distance: $y_t = \max_{xeS\ t-1(m)} FWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2													
1		0	0													
2			3													

$FWD_4(2) = 1$
$FWD_4(0) = 2$
$FWD_4(3) = 3$

Belady's Optimal Algorithm

- Replace page with maximal forward distance: $y_t = \max_{xeS\ t-1(m)} FWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2												
1		0	0	0												
2			3	1												

$FWD_4(2) = 1$
$FWD_4(0) = 2$
$FWD_4(3) = 3$

Belady's Optimal Algorithm

- Replace page with maximal forward distance: $y_t = \max_{xeS\ t\text{-}1(m)} FWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2										
1		0	0	0	0	0										
2			3	1	1	1										

Belady's Optimal Algorithm

- Replace page with maximal forward distance: $y_t = \max_{x \in S_{t-1}(m)} FWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	2									
1		0	0	0	0	0	3									
2			3	1	1	1	1									

$FWD_7(2) = 2$
$FWD_7(0) = 3$
$FWD_7(1) = 1$

Belady's Optimal Algorithm

- Replace page with maximal forward distance: $y_t = \max_{x \in S_{t-1}(m)} FWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	2	2	2	0						
1		0	0	0	0	0	3	3	3	3						
2			3	1	1	1	1	1	1	1						

$FWD_{10}(2) = \infty$
$FWD_{10}(3) = 2$
$FWD_{10}(1) = 3$

Belady's Optimal Algorithm

- Replace page with maximal forward distance: $y_t = \max_{x \in S_{t-1}(m)} FWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	2	2	2	0	0	0				
1		0	0	0	0	0	3	3	3	3	3	3				
2			3	1	1	1	1	1	1	1	1	1				

$FWD_{13}(0) = \infty$
$FWD_{13}(3) = \infty$
$FWD_{13}(1) = \infty$

Belady's Optimal Algorithm

- Replace page with maximal forward distance: $y_t = \max_{x \in S_{t-1}(m)} FWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2	2	2	2	0	0	0	0	4	4	4
1		0	0	0	0	0	3	3	3	3	3	3	6	6	6	7
2			3	1	1	1	1	1	1	1	1	1	1	1	5	5

10 page faults

- Perfect knowledge of $\varpi \Rightarrow$ perfect performance
- Impossible to implement

Least Recently Used (LRU)

- Replace page with maximal forward distance: $y_t = \max_{x \in S_{t-1}(m)} BKWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2													
1		<u>0</u>	0													
2			<u>3</u>													

$BKWD_4(2) = 3$
$BKWD_4(0) = 2$
$BKWD_4(3) = 1$

Least Recently Used (LRU)

- Replace page with maximal forward distance: $y_t = \max_{xeS\ t-1(m)} BKWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2	<u>1</u>												
1		<u>0</u>	0	0												
2			<u>3</u>	3												

$BKWD_4(2) = 3$
$BKWD_4(0) = 2$
$BKWD_4(3) = 1$

Least Recently Used (LRU)

- Replace page with maximal forward distance: $y_t = \max_{x \in S_{t-1}(m)} BKWD_t(x)$

Let page reference stream, ϖ = 203120312031 6457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2	<u>1</u>	1											
1		<u>0</u>	0	0	<u>2</u>											
2			<u>3</u>	3	3											

$BKWD_5(1) = 1$
$BKWD_5(0) = 3$
$BKWD_5(3) = 2$

Least Recently Used (LRU)

- Replace page with maximal forward distance: $y_t = \max_{xeS\ t\text{-}1(m)} \text{BKWD}_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	$\underline{2}$	2	2	$\underline{1}$	1	1										
1		$\underline{0}$	0	0	$\underline{2}$	2										
2			$\underline{3}$	3	3	$\underline{0}$										

$\text{BKWD}_6(1) = 2$

$\text{BKWD}_6(2) = 1$

$\text{BKWD}_6(3) = 3$

Least Recently Used (LRU)

- Replace page with maximal forward distance: $y_t = \max_{x \in S_{t-1}(m)} BKWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2	<u>1</u>	1	1	<u>3</u>	3	3	<u>0</u>	0	0	<u>6</u>	6	6	<u>7</u>
1		<u>0</u>	0	0	<u>2</u>	2	2	<u>1</u>	1	1	<u>3</u>	3	3	<u>4</u>	4	4
2			<u>3</u>	3	3	<u>0</u>	0	0	<u>2</u>	2	2	<u>1</u>	1	1	<u>5</u>	5

Least Recently Used (LRU)

- Replace page with maximal forward distance: $y_t = \max_{x \in S_{t-1}(m)} BKWD_t(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2	2	2	2	2	3	2	2	2	2	<u>6</u>	6	6	6
1		<u>0</u>	0	0	0	0	0	0	0	0	0	0	0	<u>4</u>	4	4
2			<u>3</u>	3	3	3	3	3	3	3	3	3	3	3	<u>5</u>	5
3				<u>1</u>	1	1	1	1	1	1	1	1	1	1	1	<u>7</u>

- Backward distance is a good predictor of forward distance -- locality

Least Frequently Used (LFU)

- Replace page with minimum use:
  $y_t = \min_{x \in S_{t-1}(m)} FREQ(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2													
1		<u>0</u>	0													
2			<u>3</u>													

$FREQ_4(2) = 1$
$FREQ_4(0) = 1$
$FREQ_4(3) = 1$

Least Frequently Used (LFU)

- Replace page with minimum use:
  $y_t = \min_{x \in S_{t-1}(m)} FREQ(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2	2												
1		<u>0</u>	0	<u>1</u>												
2			<u>3</u>	3												

$FREQ_4(2) = 1$
$FREQ_4(0) = 1$
$FREQ_4(3) = 1$

Least Frequently Used (LFU)

- Replace page with minimum use:
  $y_t = \min_{x \in S_{t-1}(m)} FREQ(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2	2	2	2										
1		<u>0</u>	0	<u>1</u>	1	1										
2			<u>3</u>	3	3	<u>0</u>										

$FREQ_6(2) = 2$
$FREQ_6(1) = 1$
$FREQ_6(3) = 1$

Least Frequently Used (LFU)

- Replace page with minimum use:
  $y_t = \min_{xeS\ t-1(m)} FREQ(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	2	2	2										
1		0	0	1	1	1										
2			3	3	3	0										

$FREQ_7(2) = ?$
$FREQ_7(1) = ?$
$FREQ_7(0) = ?$

First In First Out (FIFO)

- Replace page that has been in memory the longest: $y_t = \max_{x \in S_{t-1}(m)} AGE(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	1												
1		0	0	0												
2			3	3												

First In First Out (FIFO)

- Replace page that has been in memory the longest: $y_t = \max_{x \in S_{t-1}(m)} AGE(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	<u>2</u>	2	2													
1		<u>0</u>	0													
2			<u>3</u>													

$AGE_4(2) = 3$
$AGE_4(0) = 2$
$AGE_4(3) = 1$

First In First Out (FIFO)

- Replace page that has been in memory the longest: $y_t = \max_{x \in S_{t-1}(m)} AGE(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	1												
1		0	0	0												
2			3	3												

$AGE_4(2) = 3$
$AGE_4(0) = 2$
$AGE_4(3) = 1$

First In First Out (FIFO)

- Replace page that has been in memory the longest: $y_t = \max_{xeS\ t\text{-}1(m)} AGE(x)$

Let page reference stream, ϖ = 2031203120316457

Frame	2	0	3	1	2	0	3	1	2	0	3	1	6	4	5	7
0	2	2	2	1												
1		0	0	0												
2			3	3												

$AGE_5(1) = ?$
$AGE_5(0) = ?$
$AGE_5(3) = ?$

Belady's Anomaly

Let page reference stream, ϖ = 012301401234

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	3	3	3	4	4	4	4	4	4
1		1	1	1	0	0	0	0	0	2	2	2
2			2	2	2	1	1	1	1	1	3	3

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	0	0	0	4	4	4	4	3	3
1		1	1	1	1	1	1	0	0	0	0	4
2			2	2	2	2	2	2	1	1	1	1
3				3	3	3	3	3	3	2	2	2

- FIFO with m = 3 has 9 faults
- FIFO with m = 4 has 10 faults

Stack Algorithms

- Some algorithms are well-behaved
- Inclusion Property: Pages loaded at time t with m is also loaded at time t with m+1

LRU

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	3								
1		1	1	1								
2			2	2								

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	0								
1		1	1	1								
2			2	2								
3				3								

Stack Algorithms

- Some algorithms are well-behaved
- Inclusion Property: Pages loaded at time t with m is also loaded at time t with m+1

LRU

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	3	3							
1		1	1	1	1							
2			2	2	0							

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	0	0							
1		1	1	1	1							
2			2	2	2							
3				3	3							

Stack Algorithms

- Some algorithms are well-behaved
- Inclusion Property: Pages loaded at time t with m is also loaded at time t with m+1

LRU

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	3	3	3						
1		1	1	1	0	0						
2			2	2	2	1						

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	0	0	0						
1		1	1	1	1	1						
2			2	2	2	2						
3				3	3	3						

Stack Algorithms

- Some algorithms are well-behaved
- Inclusion Property: Pages loaded at time t with m is also loaded at time t with m+1

LRU

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	3	3	3	4					
1		1	1	1	0	0	0					
2			2	2	2	1	1					

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	0	0	0	0					
1		1	1	1	1	1	1					
2			2	2	2	2	4					
3				3	3	3	3					

Stack Algorithms

- Some algorithms are well-behaved
- Inclusion Property: Pages loaded at time t with m is also loaded at time t with m+1

LRU

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	3	3	3	4	4	4	2	2	2
1		1	1	1	0	0	0	0	0	0	3	3
2			2	2	2	1	1	1	1	1	1	4

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	0	0	0	0	0	0	0	0	4
1		1	1	1	1	1	1	1	1	1	1	1
2			2	2	2	2	4	4	4	4	3	3
3				3	3	3	3	3	3	2	2	2

Stack Algorithms

- Some algorithms are well-behaved
- Inclusion Property: Pages loaded at time t with m is also loaded at time t with m+1

FIFO

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	3	3	3	4	4	4	4	4	4
1		1	1	1	0	0	0	0	0	2	2	2
2			2	2	2	1	1	1	1	1	3	3

Frame	0	1	2	3	0	1	4	0	1	2	3	4
0	0	0	0	0	0	0	4	4	4	4	3	3
1		1	1	1	1	1	1	0	0	0	0	4
2			2	2	2	2	2	2	1	1	1	1
3				3	3	3	3	3	3	2	2	2