CS 242: Data Structures and Algorithms

First Examination (September 30th, Wednesday)

Name: _____ *Total Points: 150*

- *This exam contains 6 problems. You have 50 minutes to attempt these problems.*
- *Do not spend too much time on any problem. Read through them first and attempt them in the order that allows you to make the most progress.*
- *Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat. Except for the first problem, 10% of the grade will be reserved for clarity.*

Drill

I. (30 Points) Indicate, for each pair of expressions (A, B) in the table below, whether A is O, Θ, Ω of B. Assume that n is a variable. Your answer should be in the form of the table with "yes" or "no" written in each box.

A	B	A is $O(B)$	A is $\Theta(B)$	A is $\Omega(B)$
$n^3 2^n$	3^n			
$\lg n$	$2^{\lg \lg n}$			
$n^{\log n}$	2^n			

II. (30 Points) Use the master method to provide asymptotically tight solution to the following recurrences. You may assume, if needed, that the regularity condition holds.

(1) $T(n) = T(n/4) + \sqrt{n}$

(2) $T(n) = 2T(n/4) + \sqrt{n}$

(3) $T(n) = 4T(n/4) + \sqrt{n}$

Creative

Choose any three from the next four problems. In case you attempt all four, then make sure you specify which three should be graded.

III. (30 Points) Give an algorithm that finds out if there exist two elements x and y in a Set S (with size n) such that $\mid x - y \mid = z$. What is the worst-case runtime of your algorithm?

IV. (30 Points)

1. (10 points) What are the minimum and maximum number of elements that can be stored in a heap of height h?

2. (10 points) Is an array that is sorted in reverse order a max-heap?

3. (10 points) Explain under what circumstances a d-ary heap might be faster over a binary heap?

V. (30 Points) Describe an $O(n)$-time (the time can be average case) algorithm that, given a set S of n distinct numbers and a positive integer $k \leq n$, determines the k numbers that are closest to the median of S.

VI. (30 Points) You are given a collection of n bolts of different widths, and n corresponding nuts. You are allowed to try a nut and a bolt together, from which you can determine whether the nut is too large, too small, or an exact match for the bolt, but there is no way to compare two nuts together, or two bolts together. You are to match each bolt to its nut.

Devise an algorithm for the nuts and bolts problem that runs in time $O(n \log n)$ on the average. We will assume that it takes $O(1)$ time to try a nut and a bolt together. (20 points)